

## **Section Twelve**

### Soil Dynamics



## Three-dimensional Boundary Element Method applied to non-linear soil structure interaction.

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**Abstract:** In this work FEM and BEM combinations are considered to analyse non-linear soil-structure interactions. The first is employed to model the structure, while BEM models the two-dimensional body, using the time domain approach (TDBEM) and mass matrix boundary element technique (MMBEM). The TDBEM approach is used to model the infinity linear and often distant regions, while MMBEM is adopted to deal with the yielded soil.

### Introduction

The boundary element method (BEM) is recognised as a suitable technique for treating infinite and semi-infinite regions as the natural ground for example. This interesting property is even more evident when transient dynamic problems are considered. In this situation, the time domain boundary element formulation (TDBEM) has been successfully applied for many practical cases. Coupling of general building structures and the soil domain is an interesting application for the dynamic combination of the finite element method (FEM) and BEM. For real applications, some physical non-linearities may occur in the soil near foundation structures, requiring therefore proper analysis. As often found in the specialised literature, FEM is adopted for the non-linear region and BEM to represent the linear behaviour of the remaining domain.

This work is concerned with the development and applications of an effective BEM/FEM coupling for analysing non-linear dynamic responses of soil-structure interactions. The coupling is made by considering that the soil may yield near the structural foundations. In this situation structures are assumed elastic and will be modelled by finite elements, while the non-linear soil is modelled by mass matrix boundary elements (MMBEM) [1,2,3,4] and the elastic soil (distant regions) is modelled by time domain boundary elements (TDBEM) [5,6].

### Weighting Residuals

The weighting residual technique is employed to obtain FEM, MMBEM and TDBEM procedures. To formulate the technique we start from the dynamic equilibrium equation of an infinitesimal part of the entire domain given by:

$$\sigma_{ij,i} + b_j = \rho \ddot{u}_j + c \dot{u}_j \quad (1)$$

Multiplying eq. (1) by the weighting function called here  $u'_j$  and integrating the result over the domain  $\Omega$  results,

$$\int_{\Omega} \sigma_{ij,j} u'_j d\Omega + \int_{\Omega} b_j u'_j d\Omega = \int_{\Omega} \rho \ddot{u}_j u'_j d\Omega + \int_{\Omega} c \dot{u}_j u'_j d\Omega \quad (2)$$

where  $\Gamma$  is the boundary of the analysed body.

For simplicity the substitution of the exact fields by the approximated ones is omitted. Integrating, by parts, the first term of eq. (2), one finds:

$$\int_{\Gamma} p_j u'_j d\Gamma + \int_{\Omega} b_j u'_j d\Omega = \int_{\Omega} \rho \ddot{u}_j d\Omega + \int_{\Omega} c \dot{u}_j u'_j d\Omega + \int_{\Omega} \sigma_{ij} \varepsilon'_{ij} d\Omega \quad (3)$$

where the stress tensor symmetry and the Cauchy formula are taken into account.

Considering that  $u'_j = \delta u_j$ , in eq. (3), is the standard virtual work principle equation used to develop the finite element method. To consider discrete forces in eq. (3) the Dirac's Delta distribution can be applied for both domain or surface loads. Using usual FEM discretization and spatial integration one writes:

$$KU + C\dot{U} + M\ddot{U} = Bb + GP + IF \quad (4)$$

where  $B$ ,  $G$  and  $I$  are body force, surface force and identity matrices, respectively.

Taking into account the presence of residual stresses as  $\sigma_{ij} = \sigma_{ij}^e - \sigma_{ij}^p$  into the last term of eq. (3) and integrating it by parts one achieves:

$$\int_{\Gamma} p_j u'_j d\Gamma + \int_{\Omega} \sigma_{ij}^p \varepsilon'_{ij} d\Omega + \int_{\Omega} b_j u'_j d\Omega = \int_{\Gamma} u_j p'_j d\Gamma + \int_{\Omega} b'_j u_j d\Omega + \int_{\Omega} \rho \ddot{u}_j u'_j d\Omega + \int_{\Omega} c \dot{u}_j u'_j d\Omega \quad (5)$$

Considering the weighting function as the Kelvin fundamental solution for static problems, i.e.,  $u'_j = u_{kj}^*$ , eq.(5), turns into the usual displacement integral equation:

$$\begin{aligned} C_{kj} u_j(s,t) + \int_{\Gamma} p_{kj}^*(s,Q) u_j(Q,t) d\Gamma + \int_{\Omega} \rho u_{kj}^*(s,q) \ddot{u}_j(q,t) d\Omega + \int_{\Omega} c u_{kj}^*(s,q) \dot{u}_j(q,t) d\Omega \\ = \int_{\Gamma} u_{kj}^*(s,Q) p_j(Q,t) d\Gamma + \int_{\Omega} u_{kj}^*(s,q) b_j(q,t) d\Omega + \int_{\Omega} \varepsilon_{kij}^*(s,q) \sigma_{ij}^p(q,t) d\Omega \end{aligned} \quad (6)$$

where  $C_{kj}$  is the usual free term.

Following a well-known procedure one finds the boundary integral equation for stresses, as follows:

$$\begin{aligned} \sigma_{k\ell}(s,t) = \int_{\Gamma} \hat{\varepsilon}_{kj\ell}^*(s,Q) p_j(Q,t) d\Gamma(Q) - \int_{\Gamma} \hat{p}_{kj\ell}^*(s,Q) u_j(Q,t) d\Gamma(Q) + \\ + \int_{\Omega} \hat{\varepsilon}_{kj\ell}^*(s,q) b_j(q,t) d\Omega(q) - \int_{\Omega} c \hat{\varepsilon}_{kj\ell}^*(s,q) \dot{u}_j(q,t) d\Omega(q) + \\ - \int_{\Omega} \rho \hat{\varepsilon}_{kj\ell}^*(s,q) \ddot{u}_j(q,t) d\Omega(q) + \int_{\Omega} \hat{\varepsilon}_{kij\ell}^*(s,q) \sigma_{ij}^*(q,t) d\Omega(q) + \hat{g}_{kij\ell}^* \sigma_{ij}^p(s,t) \end{aligned} \quad (7)$$

From usual the spatial BEM discretization one finds:

$$H U(t) + C \dot{U}(t) + M \ddot{U}(t) = G P(t) + B b(t) + Q \sigma^P(t) \tag{8}$$

$$\sigma(t) = G' P(t) - H' U(t) + B' b(t) - M' \ddot{U}(t) - C' \dot{U}(t) + Q' \sigma^n(t) \tag{9}$$

From equation (3), without dumping, integrating by parts the last term and all equation regarding time, results the following TDBEM integral equation.

$$\begin{aligned} C_{kj} \int_0^t u_j(s, \tau) f(t - \tau) d\tau &= \int_0^t \int_{\Gamma} u_{kj}^*(Q, t - \tau; s / f) p_j(Q, \tau) d\Gamma d\tau - \\ &- \int_0^t \int_{\Gamma} p_{kj}^*(Q, t - \tau; s / f) u_j(Q, \tau) d\Gamma d\tau + \int_0^t \int_{\Omega} b_j(q, \tau) u_{kj}^*(q, t - \tau; s / f) d\Omega d\tau + \\ &+ \int_{\Omega} \rho u_{kj}^*(q, t; s / f) u_j(q, 0) d\Omega - \int_{\Omega} \rho \dot{u}_{kj}^*(q, t; s / f) u_j(q, 0) d\Omega \end{aligned} \tag{10}$$

where f(t) is the time behaviour of the fundamental load of the Stokes fundamental solution.

Adopting the usual TDBEM time and space discretizations results:

$$H_1 U_{n_1} = G_1 U_{n_1} + F_{n_1} \tag{11}$$

where 1 is the first time step and n<sub>1</sub> the actual one.

Bearing in mind that Newmark's time integration scheme is used to integrate eq. (4) and Houbolt's scheme for eqs (8) and (9), the resulting time marching equations together with eq. (11) can be written for a prescribed time step as:

$$H^{(i)} U^{(i)} = G^{(i)} U^{(i)} + F^{(i)} + F^P \tag{12}$$

where F<sup>P</sup> comes from eq. (9).

It is possible to apply the sub-region technique for any number of sub-regions represented by an equation as (12). The non-linear procedure is completed remembering that F<sup>P</sup> is calculated by an iterative procedure taking into account the global sub-region equation.

### Examples

#### Cylindrical Cavity

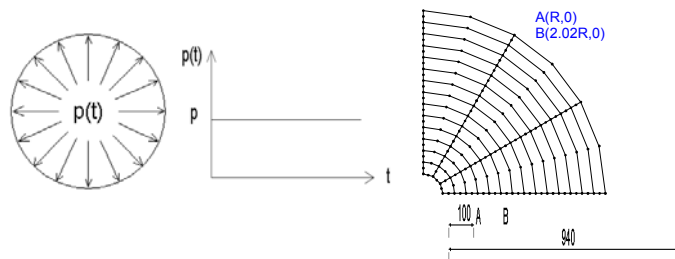


Figure 1. Geometry, external loading and discretization

A cylindrical cavity under an internal pressures, as depicted in Fig. 1, is analysed and discussed. The physical properties are: Young’s modulus  $E = 94,6769\text{ksi}$ ; Poisson’s ration  $\nu = 0,2308$ ; and the mass density  $\rho = 3,5 \text{ slug/ft}^3$ .

After adopting the time step  $\Delta t=2\text{s}$ , the points are  $A$  and  $B$  were selected to conduct the analysis. The results compare very well with reference [3] values (Fig. 2). The Drucker Prager yielding criterion is adopted with the following parameters:  $c = 0.9 \text{ ksi}$ ,  $c = 0,70 \text{ ksi}$ ,  $c = 0,50 \text{ ksi}$  and  $\phi = 30^\circ$ .

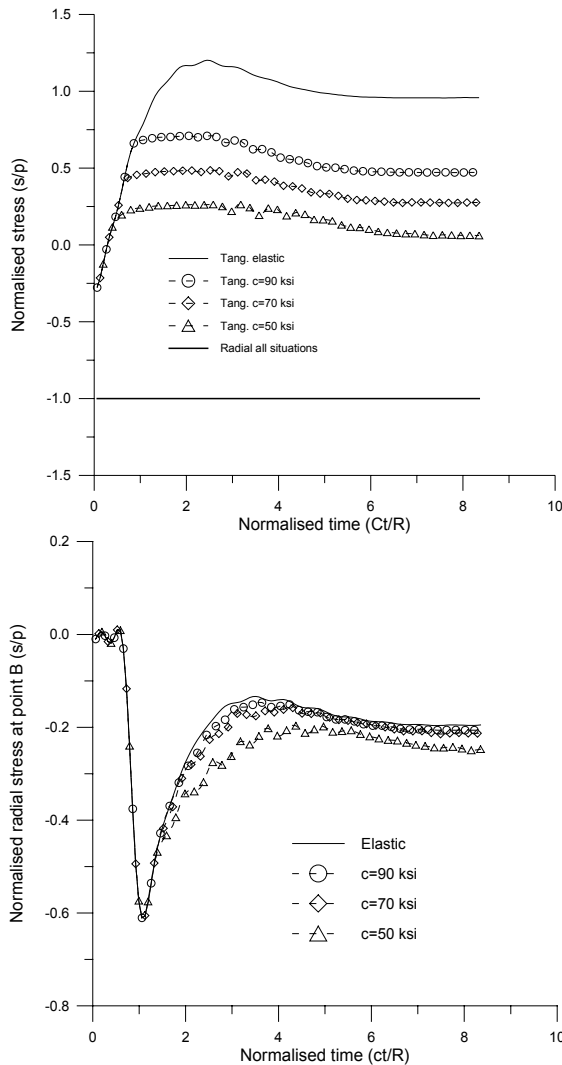


Figure 2. Radial stress components for points A and B

**Simple stretched rod**

A simple stretched rod is subjected to a suddenly applied load as depicted in Fig. 3, in which the physical parameters are also given. Two discretizations are adopted as

shown in Fig. 4. The adopted time step for MMBEM is  $\Delta t_{mmbem} = 0.00012s$ , while for the Coupling (MMBEM/TDBEM/FEM) is  $\Delta t_{coupling} = 0.0002s$ . This difference is due to the stability conditions for TDBEM when dealing with finite bodies. The upper end displacements are illustrated in Fig. 5.

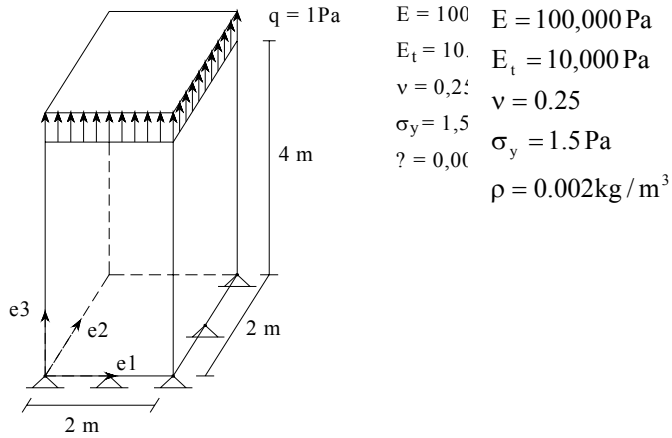


Figure 3. Geometry, loading and physical parameters

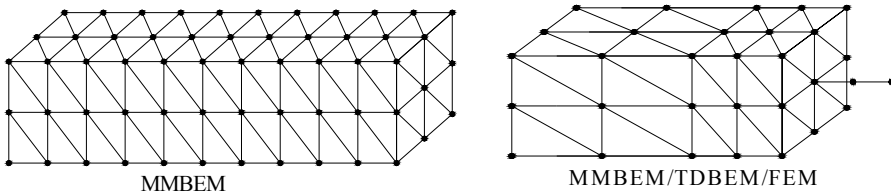


Figure 4. Discretizations

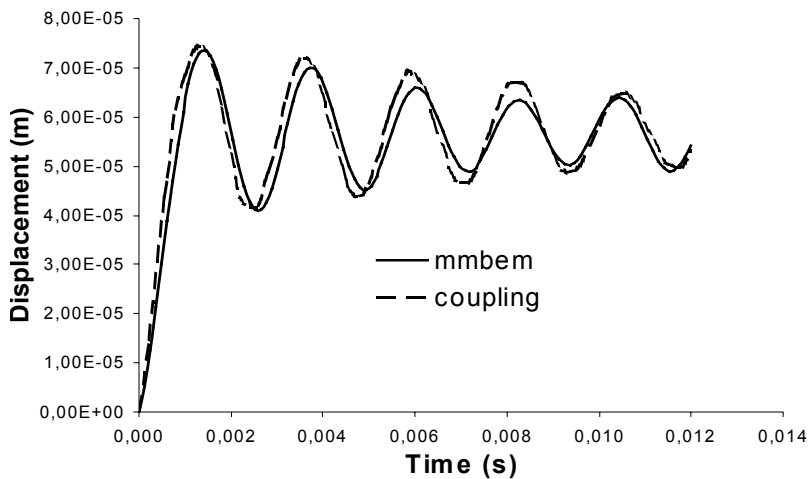


Figure 5. Displacement of the loaded surface.

The results are in almost perfect concordance, despite the difference of the adopted discretizations.

### **Conclusions**

The BEM/TDBEM/FEM coupling has been successfully implemented. The first example has confirmed that the MMBEM performance for non-linear analysis is excellent. The applicability of the three-dimensional coupling for general problems is demonstrated by the example two. This example is a rather difficult problem for TDBEM analysis, demonstrating therefore that the technique is well implemented, accurate and stable. Due to the degree of difficulty of example two, it is obvious that the resulting formulation is a powerful tool for general soil-structure interaction problems and others reinforced medium cases.

### **Acknowledgements:**

The authors would like to thanks FAPESP - São Paulo State Research Foundation - for supporting this research.

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## **Dynamic Interaction between two close structures on a poroelastic soil**

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**Keywords** Dynamic interaction, structure-soil-structure interaction, poroelastic soil, FEM – BEM coupling

**Abstract.** The dynamic interaction between two close three-dimensional structures, through the underlying or surrounding poroelastic soil, is investigated by coupling finite and boundary elements. A three dimensional problem is considered, using a finite element beam formulation for the structures coupled with a boundary element formulation for the soil. The material of the structures is linear elastic with structural damping. The foundation soil is a half-space which is a homogeneous, isotropic and viscoelastic or poroelastic material. The structures simulated are two adjacent towers supported by two identical square foundations embedded in a soil stratum. A harmonic force is applied on one of the structures. On the base of the numerical studies, the effects of the location of the load and the effects of distance between two foundations are examined.

### **Introduction**

The dynamic interaction between adjacent structures on viscoelastic soil have received considerable attention in recent years. Several methods to couple structure-soil –structure have been successfully applied in this area.

The dynamic behavior of poroelastic soil has been also studied recently, with important contributions to a proper understanding of his influence in the interaction between soil and structure. Several 2D and 3D studies have been presented computing compliances for rigid foundations resting on or embedded in poroelastic soils.

The aim of this article is to study the interaction between adjacent structures through the soil, considering the soil as a uniform two-phase fluid-saturated poroelastic medium.

**Basic formulations.**

**Finite element formulations.** The structures have been discretized by 3D beam-type finite elements with distributed mass.

The material is linear elastic with material damping of hysteretic type.

The dynamics equations of the motion are formulated in the frequency domain.

The algebraic system of equation is given by

$$\begin{matrix} \acute{e}S_{ss} & S_{sb} \grave{u} \acute{e}u^t \grave{u} & \acute{e} P_s \grave{u} \\ \acute{e}S_{bs} & S_{bb} \acute{u} \acute{e}u^t \acute{u} & \acute{e} P_{bT} \acute{u} \end{matrix} = \begin{matrix} \acute{e} P_s \grave{u} \\ \acute{e} P_{bT} \acute{u} \end{matrix} \tag{1}$$

where *s* and *b* denote the nodes non-connected and connected to the soil, respectively. The amplitudes of the loads are represented by *P*, and  $P_{bT} = P_b + P_{b0}$  is the summation of the interaction forces with the soil and the directly-applied forces on connected nodes. The total displacements are denoted by the superscript *t*.

When the coupled zones are a viscoelastic zone (foundation) and a poroelastic zone the equilibrium conditions along the interface is:

$$\begin{matrix} \text{- impervious interface: } P_b = -E_1 \cdot T^p = -E_1 \cdot (t^p + \mathbf{t}) = -E_1 \cdot (t^p + \mathbf{f} \cdot p^p) \\ \text{- pervious interface: } P_b = -E_1 \cdot T^p = -E_1 \cdot t^p \end{matrix}$$

where  $T^p$  denote total traction on the poroelastic medium ,  $p^p$  the pore pressure and  $\phi$  the porosity, and  $E_1$  the transformation matrix between forces and tractions. Both conditions are included in the expression

$$P_b = -E \cdot t_c \tag{2}$$

where  $P_b$  denotes the force on the connect node of the beam. And  $t_c$  denotes the tractions over the interface of the connection

**Boundary element formulations.** 3D-quadratic elements have been used to model the soil surface. The classical system of linear equations can be written as

$$\begin{matrix} \acute{e}H_{gg} & H_{gc} \grave{u} \acute{e}u^t \grave{u} & \acute{e}G_{gg} & G_{gc} \grave{u} \acute{e}t_g \grave{u} \\ \acute{e}H_{cg} & H_{cc} \acute{u} \acute{e}u^t \acute{u} & \acute{e}G_{cg} & G_{cc} \acute{u} \acute{e}t_c \acute{u} \end{matrix} = \begin{matrix} \acute{e}G_{gg} & G_{gc} \grave{u} \acute{e}t_g \grave{u} \\ \acute{e}G_{cg} & G_{cc} \acute{u} \acute{e}t_c \acute{u} \end{matrix} \tag{3}$$

where *u* and *t* are displacements and tractions, *H* and *G* the standard BEM matrices. The subscript *g* and *c* denote the non-connected nodes and the connected nodes to the structure, respectively.

Using the compatibility conditions at the interface:

- impervious interface:  $u_{interface} = u^p$  and  $u_{interface}^n = u^m = w$
- pervious interface:  $u_{interface} = u^p$

where the superscript n denote normal displacements to the interface, and the rigid-solid conditions:

$$u_c^t = D . u_b^t \tag{4}$$

**Coupling.** A set of coupled finite and boundary elements equations can be written as follows:

$$\begin{matrix}
 \hat{e} H_{gg} & - G_{gc} & \{H_{gc} \times D\} & \mathbf{0} & \hat{u} & \hat{e} u_g^t & \hat{u} & \hat{e} G_{gg} & t_g & \hat{u} \\
 \hat{e} H_{cg} & - \hat{G}_{cc} & \{\hat{H}_{cc} \times D\} & \mathbf{0} & \hat{u} & \hat{e} t_c & \hat{u} & \hat{e} G_{cg} & t_g & \hat{u} \\
 \hat{e} \mathbf{0} & E & S_{bb} & S_{bs} & \hat{u} & \hat{e} u_b^t & \hat{u} & \hat{e} P_{b0} & \hat{u} \\
 \hat{e} \mathbf{0} & \mathbf{0} & S_{sb} & S_{ss} & \hat{u} & \hat{e} u_s^t & \hat{u} & \hat{e} P_s & \hat{u}
 \end{matrix} = \tag{5}$$

**Numerical studies**

In order to study the interaction effects between three-dimensional structures through the underlying or surrounding soil, a simple structure-soil-structure system is chosen, simulating two adjacent towers, taken from Wang and Schmid [7]. A load  $P=1.0 \times 10^9$  N is applied at the bottom of the first tower in the direction of the alignment of the towers.

The structures have distributed masses and a circular cross section. The foundations are the embedded at a depth of 2 m. and have a section of 2m x 2m. The material of structures has a Young’s modulus  $E= 2.6 \cdot 10^{10}$  N/m<sup>2</sup>, a Poisson’s ratio  $\nu=0.33$ , a mass density of  $\rho=7200$  kg/m<sup>3</sup>, and a damping factor  $\beta=0.05$ . Four finite beam elements are used in each tower.

The viscoelastic soil properties ,corresponding to a dense sand, are taken from Kassir and Xu [5]: shear modulus of the skeleton  $G= 3.2175 \cdot 10^7$  N/m<sup>2</sup>, Poisson’s ratio  $\nu=0.25$ , porosity  $\phi = 0.35$ , density of the solid material  $\rho_f = 1000$  kg/m<sup>3</sup>, apparent added density  $\rho_a = 0$ ; permeability  $k = 10^{-4}$  m.s<sup>-1</sup> (corresponding to a seepage force constant  $b = 1.1986 \times 10^7$  N.s/m<sup>4</sup>); and Biot’s constants  $Q = 4.61 \times 10^8$  N/m<sup>2</sup> and  $R = 2.4823 \times 10^8$  N/m<sup>2</sup>. All elements used in the neighbourhood of the foundations are selected smaller then one half of the shear wave length.

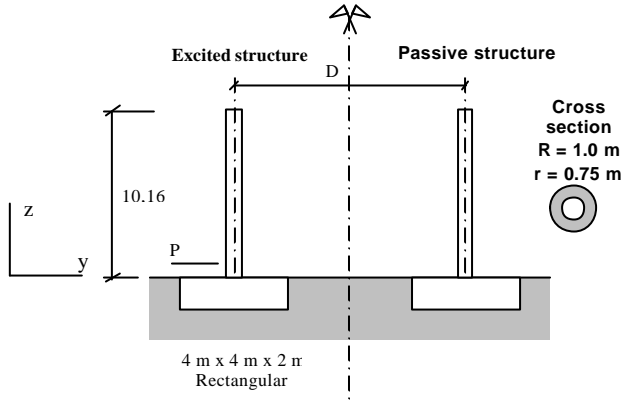


Fig 1. Model scheme of the towers

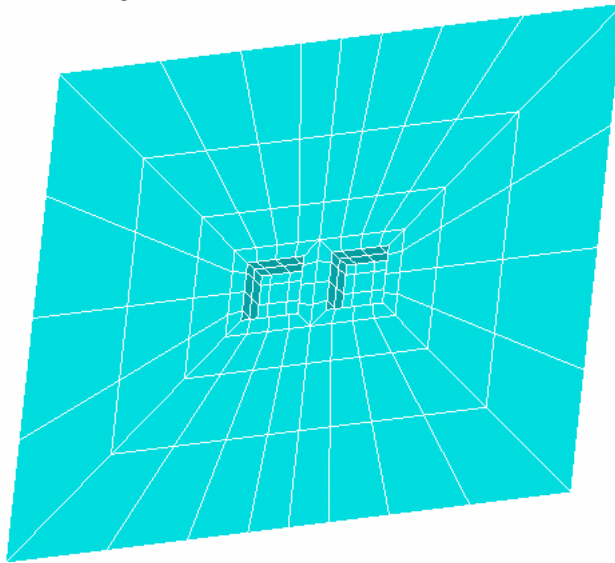


Fig 2. BEM model of the soil and foundations

The relative position of the two foundations is defined by the distance between the two centres and is called  $D$ . For the purpose of examining the influence of the distance between two foundations, the displacements  $u_x$  and  $u_z$  of the bases of the towers are shown in fig. 3, with two different distances:  $D = 6\text{ m}$ . and  $D = 10\text{ m}$ . The results are plotted versus the frequency normalized by the first natural frequency of the tower.

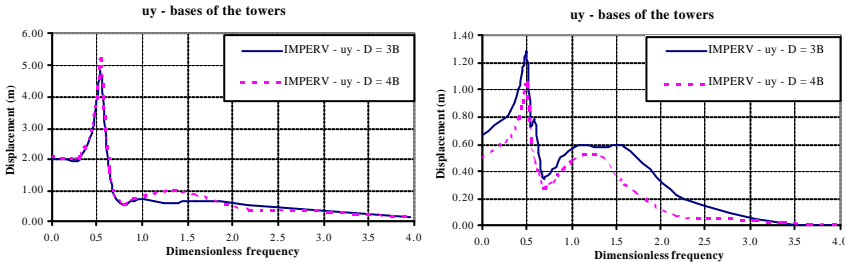


Fig.5. Effects of distance between foundations. Impervious interface. Distances  $D = 3B$  (6 m) and  $D = 4B$  (10 m).

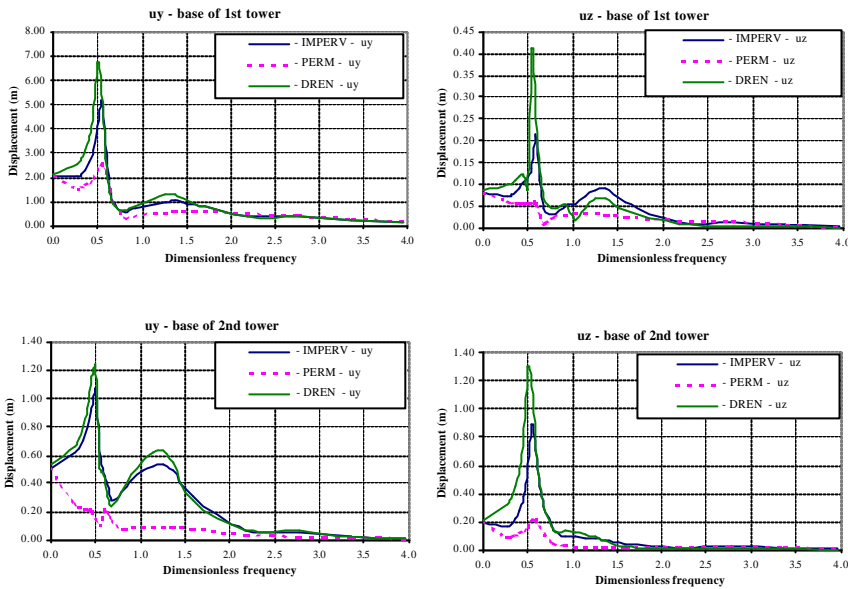


Fig 4. Movements of the bases of the towers. Different models of the soil.  $D = 4B$

The distance has a great influence on the interaction, and the induced effect in the passive structure decreases quickly when distance increases. The excited structure is not significantly affected by the distance.

To show the influence of the conditions at the interface, the displacements in the cases of impervious and pervious conditions are presented in fig. 4. They are also compared with the response of the structure with a equivalent drained soil (viscoelastic soil with identical shear modulus and Poisson's modulus to the proelastic soil).

The differences are important, specially between impervious and pervious interfaces, and they are caused by the equivalent damping effects of the drainage of the water. Drained soil has a similar response to the impervious one, but more pronounced in the neighbourhood of the resonant frequency of the structures.

### **Acknowledgment**

This work is partially supported by the Comisión Interministerial de Ciencia y Tecnología of Spain under research project DPI 2001-2377-CO2-01. The financial support is gratefully acknowledged.

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## Dynamic Stiffness of Piles in Uniform Soils

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**Keywords:** Boundary elements, wave propagation, dynamics, piles, soil-structure interaction

**Abstract.** A boundary element technique for the computation of dynamic stiffness of piles and pile groups is presented in this paper. The approach is based on the boundary element formulation for time harmonic viscoelastic media. Quadratic elements are used for piles and free surface discretization. Results are obtained for single piles and pile groups. The computed values are compared with others existing in the literature. The agreement is very good.

### Introduction

Boundary Element Methods (BEM) based on boundary integral equations are very well suited for dynamic soil-structure interaction problems. They have become a very extended approach for the solution of this type of problems due to their ability to represent boundless regions in a natural way. The radiation of waves towards infinity is automatically included in the model, which is based on an integral representation valid for internal and external regions. An extensive treatment of these BE applications can be found in [1].

### Dynamic Stiffness

The dynamic stiffness matrix of a foundation relates the vector of forces (and moments)  $\mathbf{R}$ , applied to the foundation and the resulting vector of displacements (and rotations)  $\mathbf{u}$ .

$$\mathbf{R} = \mathbf{K} \mathbf{u} \quad (1)$$

The dynamic stiffness terms for a time harmonic excitation, are functions of frequency  $\omega$  and are usually written as:

$$K_{ij} = K_{0ij}(k_{ij} + ia_0 c_{ij}) \tag{2}$$

where  $K_{0ij}$  is the static value of the  $ij$  stiffness component,  $k_{ij}$  and  $c_{ij}$  are the frequency dependent dynamic stiffness and damping coefficients, respectively,  $a_0$  is the dimensionless frequency

$$a_0 = \frac{2\omega R}{c_s} \tag{3}$$

and  $c_s$  the soil shear-wave velocity

$$c_s = \sqrt{\frac{G_s}{\rho_s}} \tag{4}$$

The coordinates definition for a pile group can be seen in Fig.1.

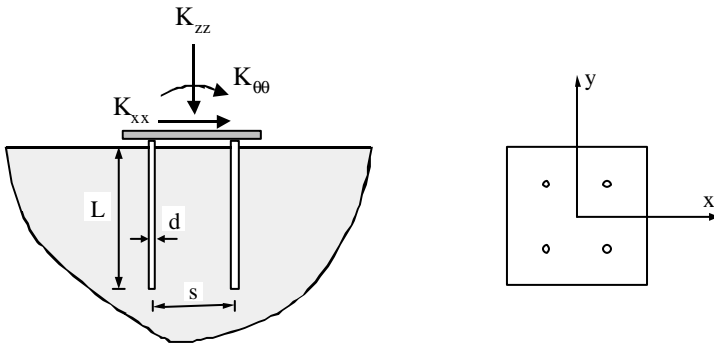


Figure 1 . Pile group and coordinates definition

Due to the use of a fundamental solution corresponding to the complete space, not only the soil-foundation interface, but also the soil free-surface, should be discretized. However, in practice only a small region around the foundation has to be included in the model since there is a small effect of the free-surface far away from the foundation on the computed values of the stiffness coefficients. Some authors have used the half-space fundamental solution for the BE analysis of problems involving a half-space. In such a case, the soil free-surface is

automatically taken into account and no discretization of this part of the boundary is required. However, there is a price paid for the simplification. Since there is no closed form expression for the half-space fundamental solution, which depends on boundless integrals, rather involved approximate procedures are required for its evaluation. The author's experience shows that the use of a full-space fundamental solution is very simple and produces accurate results with discretizations of the free-surface restricted to a rather small region around the foundation. In the case of piles, preliminary results show that an accurate representation of the dynamic behavior is obtained with a free soil discretization extending up to a distance from the pile top equal to its length,

### Boundary element model and numerical results

**Single pile in uniform half-space.** This is a fundamental dynamic soil-structure interaction problem for which some other numerical results exist. In the present analysis, both the pile and the surrounding soil are assumed to be uniform viscoelastic media with a 5% internal damping. The ratio between the material modulae is:  $E_s/E_p=10^{-2}$ ,  $E_s$  being the elastic modulus of the soil and  $E_p$  that of the pile. The ratio between the densities is:  $\mathbf{r}_s/\mathbf{r}_p = 0.7$ ; and the Poisson's ratio:  $\mathbf{n}_s = 0.4$  and  $\mathbf{n}_p = 0.25$ , respectively. The pile aspect ratio is  $L/R = 40$ , where L is the length and R the radius (see Figs. 2 and 3).

The boundary element discretization is as shown in Fig.2, where all the elements are nine-node quadratic elements. Time harmonic displacements along the vertical axis z, a horizontal axis, as well as a rotation perpendicular to the pile, are prescribed at the pile top surface. Compatibility and equilibrium conditions are prescribed on the soil-pile interface. The computed values of the vertical and horizontal stiffness coefficients are shown in Figs. 4 and 5, respectively. These values are in good agreement with those presented in [2].

**Pile group in uniform half-space.** Figure 3 shows the boundary element discretization for the problem of Fig.1. The top surface of the four piles are assumed to be connected to a rigid body. Once again, horizontal, vertical, and rotational unit motions are prescribed at the top to compute the corresponding stiffness coefficients. The computed values are compared in Figs. 4 and 5 with those obtained in [2]. There is a good agreement between both sets of results.

### Conclusion

A robust BE approach for the dynamics analysis of piles and pile groups

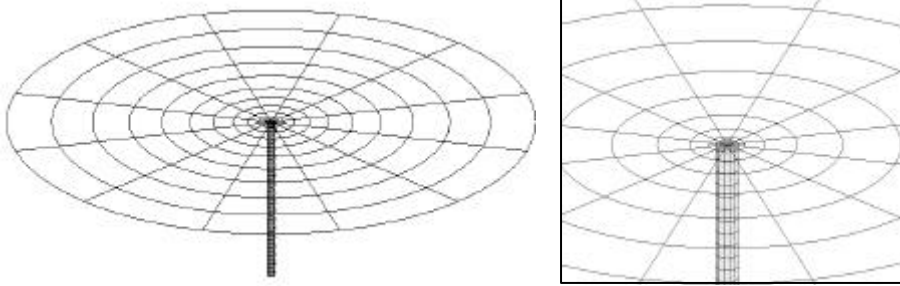


Figure2. Boundary discretization for single pile in a half-space

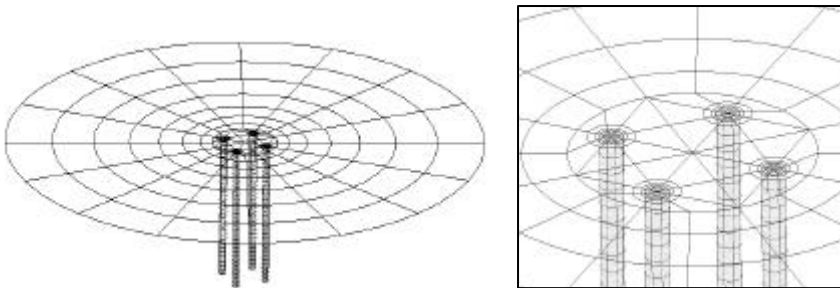


Figure3. Boundary discretization for 2 X 2 pile group in a half-space

has been presented in this paper. Different to other approaches the pile are assumed to be linear viscoelastic solids and the compatibility and equilibrium conditions between piles and the soil are imposed rigorously.

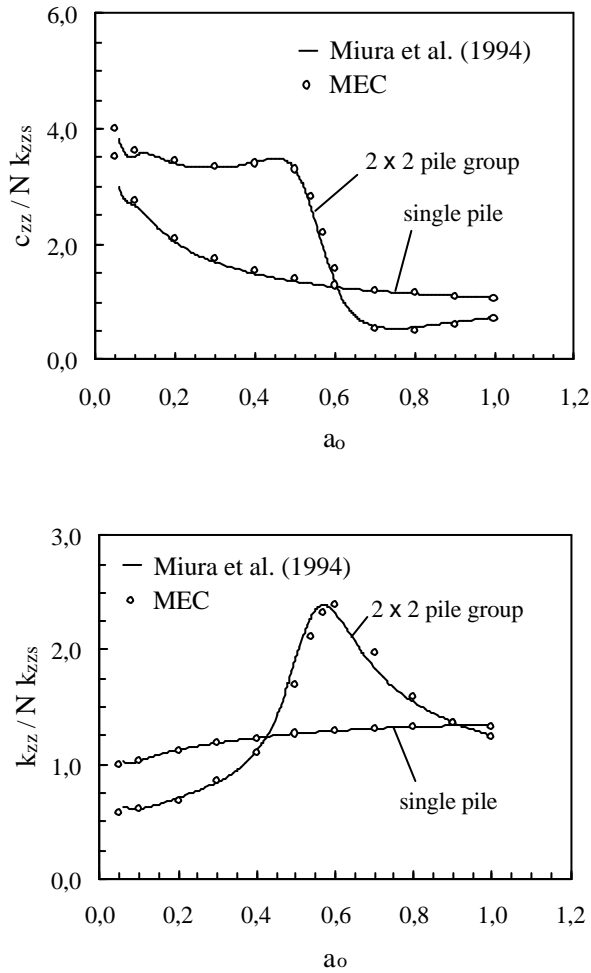


Figure 4. Vertical dynamic damping and stiffness coefficient for single pile and 2 X 2 pile group

**Acknowledgements**

The authors would like to acknowledge the financial support of the Ministerio de Ciencia y Tecnología of Spain (Projects DPI2000-1217-C02-01 and DPI2001-2377-C02-01).

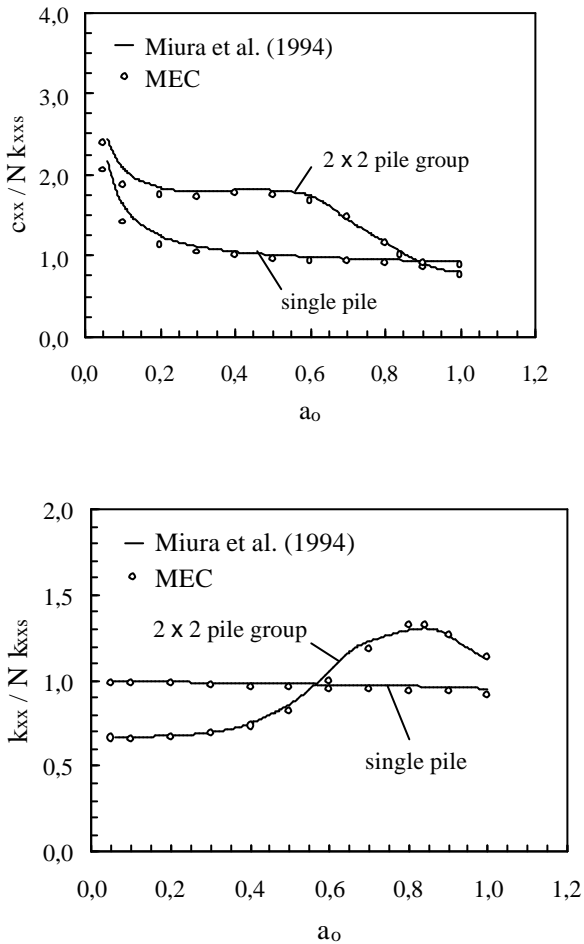


Figure 5. Horizontal dynamic damping and stiffness coefficient for single pile and 2 X 2 pile group

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